ANNUAL REPORT ON FY97 ONR SPONSORED RESEARCH

EXCITATION, GENERATION AND PROPAGATION OF SHORT CAPILLARY WAVES INTERACTING WITH LONGER WAVES

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LONG-TERM GOAL

The first goal of this effort is to achieve a better understanding of the role of capillary waves in ocean surface phenomena. A second goal is to investigate the propagation of ocean waves into shallow water, as on beaches and reefs, and the generation of capillary waves from these.

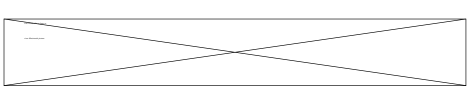
OBJECTIVES

Objectives of this research are to demonstrate a physical model and a practical computational model for investigating capillary wave interactions with longer waves and to apply this to applications, such as generation and spectra in both wavenumber and frequency.
Further objectives are to model nonlinear waves in shallow water, including the generation of capillary waves by these.

APPROACH

Capillary wave interactions with longer waves: The model developed is described in ref(1). Applications were made in ref(1) to waves in one dimension (when I refer to waves in “one dimension” I mean one surface dimenstion; when I refer to waves in two dimensions, I mean waves in two surface dimensions). Applications to waves in two dimensions have been made during the current year.

The model used begins with a conventional formulation of irrotational surface wave hydrodynamics. The displacement of the water surface is Fourier expanded in a rectangular box of sides L_x and L_y :

 (1)

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In this equation **k** is a wave number and $\;_{\mathrm{k}}$ is the linear wave frequency. Time-dependent equations for the Fourier amplitudes are deduced from the Navier-Stokes equations (we used a Hamiltonian method, which leads to the same results). These have the form

$$\dot{a}_{\mathbf{k}} + e_1 [b\, a_{\mathbf{k}}] = e_3\, T_3(\mathbf{a};\mathbf{k}) + e_4\, T_4(\mathbf{a};\mathbf{k}) + \ldots \qquad (2)$$

Here

$$\begin{aligned} T_3(\mathbf{a};\mathbf{k}) = -\frac{i}{8V_k}\sum_{l,p}\frac{k}{lp}\{ & d_{\mathbf{k-l-p}}\,\Gamma(\mathbf{k},\mathbf{p},\mathbf{l})\, a_{\mathbf{l}}\, a_{\mathbf{p}}\, e^{i(w_k - w_l - w_p)t} \\ & + 2 d_{\mathbf{k+l-p}}\,\Gamma(\mathbf{p},\mathbf{k},\mathbf{l})\, a_{\mathbf{p}}\, a_{\mathbf{l}}^{*}\, e^{i(w_k + w_l - w_p)t} \\ & + d_{\mathbf{k+l+p}}\, h(\mathbf{l},\mathbf{p},\mathbf{k})\, a_{\mathbf{l}}^{*}\, a_{\mathbf{p}}^{*}\, e^{i(w_k + w_l + w_p)t}\}. \end{aligned} \qquad (3)$$

The coefficients   and h are given, for example, in ref(1). The quantity T_4 in (2) is similar to (3), but contains sums of products of three a's. The quantity   represents the decay rate due to viscosity. The coefficients   and   are each 0 or 1, depending upon whether we wish to keep the corresponding term in our calculation.

The term T_3 is called the "three wave" term since it explicitly represents three interacting waves. The term T_4 is called the "four wave" term because it explicitly exhibits four wave interactions. Higher order terms are neglected in (2). A "resonance" occurs when the argument of a time dependent exponential vanishes, such as

$$w_k \pm w_l - w_p = 0. \qquad (4)$$

Three wave resonances occur in T_3 and 4-wave resonances occur in T_4. There are no 3-wave resonances for purely gravity waves. For capillary waves surface tension leads to 3-wave resonances. Based on our analyses in ref (1), for capillary wave-short gravity wave interactions the term T_3 therefore appears to be dominant in (2). Most published analyses of capillary waves have, in fact, omitted the 4-wave term T_4 from (2).

Terms in (3) with rapidly oscillating exponentials substantially increase computing times. To mitigate this problem I introduced a transformation of amplitude variables (a "canonical transformation"),

$$a_{\mathbf{k}} \to A_{\mathbf{k}}, \qquad (5)$$

that removes terms with rapidly oscillating exponentials from (3). After integrating the "reduced" set of equations (2), it is necessary to integrate a second set of equations to return to the physical amplitudes $a_{\mathbf{k}}$. This takes proper account of the effects from the terms that were removed from (3).

In the transform space of the $A_{\mathbf{k}}$ variables the terms that oscillated rapidly in time become the most slowly varying. Factors of 10 or greater reduction in computing time were achieved with this technique.

Gravity waves in shallow water: The dynamic properties of gravity waves in shallow water are dominated by three wave terms, as in (3). This permitted me to easily adapt my computational models to the description of waves in shallow water. Two models have been developed. The first and simplest describes waves in water of constant depth. The second model considers variable depth. This begins with a set of WKB linear wave solutions for the specified depth profile. These WKB modes are coupled by the nonlinear interactions.

WORK COMPLETED

Capillary waves: Application of the models described above have been made to wave systems in two dimensions:
The evolution of a wind wave field into the capillary regime has been investigated.
Fine scale roughness on the surface of gravity waves has been studied.
Frequency-wave number spectra are being investigated. The autocorrelation function

$$< z(\mathbf{x},t) z(\mathbf{x}+\mathbf{r},t+t) > \tag{6}$$

has been calculated as an ensemble average. From this we plan to deduce spectra, as has been done in ref (2) for observed wave systems.
Waves in shallow water: The temporal evolution of waves propagating into shallow water has been investigated.

RESULTS

We show in Fig. (1) a sample of the surface roughness for a wind wave field. This is a cut along the x-axis of waves in two dimesnions. The heavy shows suface displacement z as a function of x at a fixed y. The light line shows the corresponding wave slope filtered to show waves of 2cm and less wavelength. The strong effects of modulation are evident.

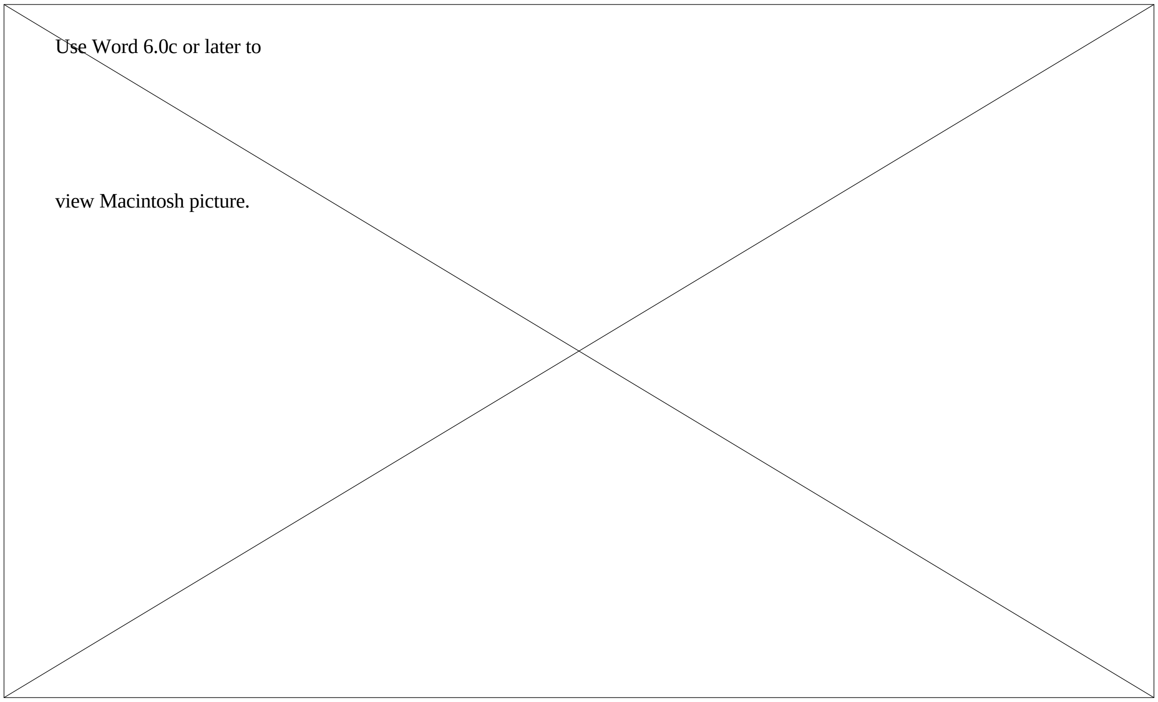
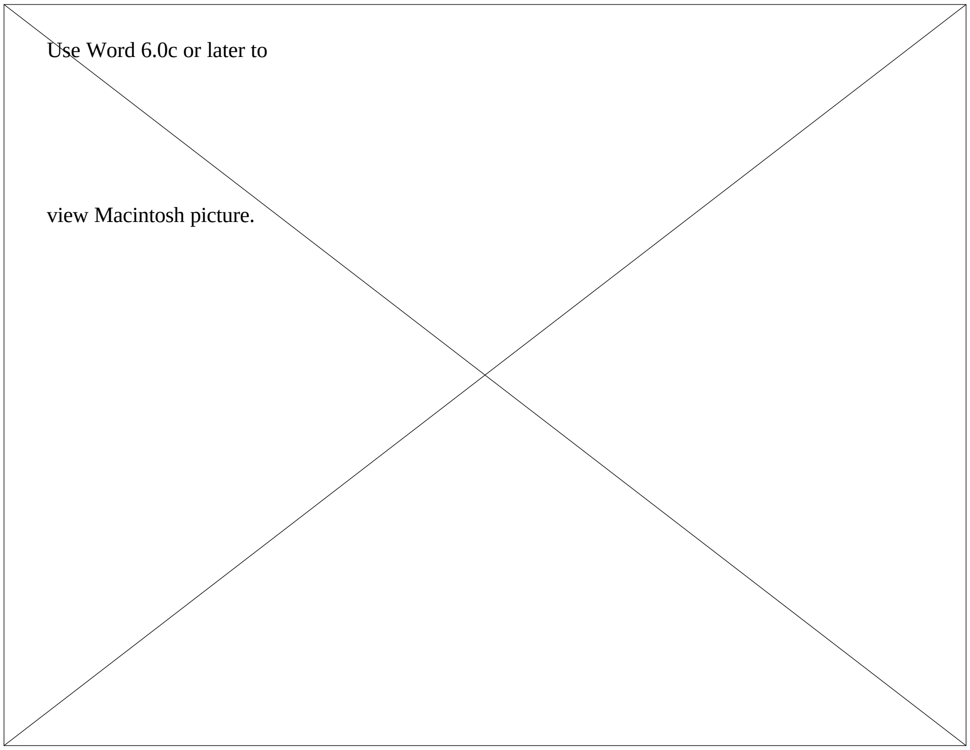


Fig.(1) Surface roughness and capillary modulation.

We show in Fig.(2) the autocorrelation function $< a_{\mathbf{k}}(t)a_{\mathbf{k}}(t+t) >$ as a function of τ for k=0.3 rad/cm (solid curve) and k=3 rad/cm (dashed curve).



In Fig.(3) we illustrate the propagation of a wave packet from deep water into water 2 m deep. The calculation was initialized with the wave packet in deep water.

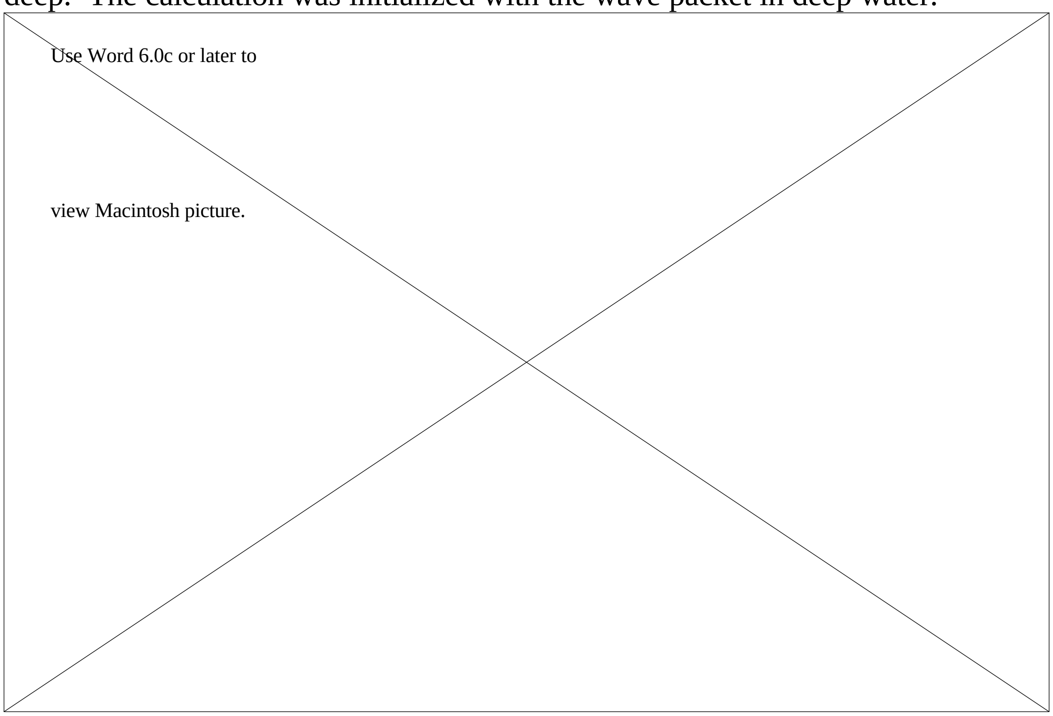


Fig.(3) Surface displacement for a wave packet in shallow water.

IMPACT/APPLICATION

Capillary waves: The canonical transformation technique that we have introduced is, to our knowledge, new. It permits one to achieve an order of magnitude, or more, reduction in computing time. Although this has been applied only to interactions of capillary waves, we think the method has more general applicability.

Capillary waves are important for remote sensing and for understanding breaking of gravity waves. Our model provides a relatively simple way to study capillary wave generation and modulation by gravity waves.
Nonlinear interactions modify the linear wave dispersion relation. Our calculation of the autocorrelation function provides a means for investigating this.

Gravity waves in shallow water: The steepening and onset of breaking of waves moving into shallow water have an impact on coastal engineering and other coastal operations.

RELATED PROJECTS

I have benefited in this work from interactions with several investigators doing related research. This includes M. Longuet-Higgins, B. Jaehne, K. Melville, and R. Guza.

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